## MATHEMATICAL MODELING OF THERMAL REGIMES OF SHS COMPACTION

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## Results of mathematical modeling of SHS compaction are discussed.

The utility and advantages of mathematical modeling in developing new processes and designing equipment are apparent. This also refers to SHS technologies of manufacturing products from high-melting materials using methods of extrusion and pressing. The processes of SHS compaction proceed under extreme temperature conditions with a wide characteristic temperature range – from the combustion temperature  $T_c$  (2000-3000°C) to the ambient temperature. In this connection the level of thermal gradients determines to a large extent the quality of the finished product. Under the conditions of high-temperature regimes of deformation, powder materials behave as viscoductile solids whose deformation occurs mainly on account of a flowing component which is either a more ductile material of powder composition (if there is a solid-phase interaction) or a molten metal or a solution on its base (in the case of a liquid-phase interaction). The presence of the flowing component is determined by the temperature of the compound: with decreasing temperature due to intense heat removal the ability of the material to viscous flow sharply decreases, the material freezes and loses its ductile properties.

Thermal models of SHS pressing and SHS extrusion have been formulated for studying the processes of sharp cooling of combustion products, which involves a loss of the ductile properties of the material. On the basis of these models the temperature fields are investigated which arise both in the material of the specimen and in the elements of press attachments. Such an investigation, on the one hand, makes it possible to determine the optimal parameters of the process and to predict experimental results and, on the other hand, offers possibilities for calculating press attachments with due account of the real conditions under which the processes of SHS compaction proceed.

Thermal Model of SHS Extrusion. Briefly, the essence of the method of SHS extrusion is in the following [1, 2]. A powder compact of a mixture of various substances is placed into a mold which is in the working container of a press. With the aid of a firing device a combustion wave is initiated over the specimen. After the combustion wave has passed over the material, a high-melting compound forms which is quickly extruded through the hole of a forming die by the plunger of the press. As a result, in tens of seconds at fairly low pressures (P < 1000 MPa) we obtain a practically nonporous long-dimensional product of the required shape and dimensions.

The development of the process of extrusion depends on the regime factors, the external conditions, the intrinsic properties of the material, and the geometry of the press attachments and their characteristics. These influences should be taken into account by the parameters of the model. Among these are the thermophysical parameters of the specimen and the envelope, which are calculated for some mean characteristic temperature for each stage and the mean porosity of the specimen, the thermophysical characteristics of the press attachments and their geometry as well as the geometry of the specimen. Much attention has been given to the calculation of parameters and criteria governing thermal boundary conditions. The model contains technological parameters such as the velocity of the plunger of the press, the delay time, etc. We will call attention to the following effective characteristics: the combustion temperature of the material, the burning rate, and the viable temperature of the material which are measured directly in a real experimental plant. With a temperature above the viable temperature the material displays the ability to plastic deformation, below this temperature it freezes. Thus, the model does not contain the so-called free parameters which are frequently used to fit the model to the experiment. This enables us to employ the model for various experimental situations and to directly compare theory with experiment.

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Fig. 1. Geometric region in which the equations are solved: 1) envelope; 2) specimen.

For investigating the thermal regimes a model of heat transfer in the specimen and the envelope is formulated (Fig. 1). The investigated system is symmetric about the angular coordinate, and the temperature may be thought of as a function of two coordinates (vertical z and radial r) and the time t. The mathematical model includes a system of differential equations for the envelope

$$\frac{\partial T_1}{\partial t} = a_1 \nabla^2 T_1 = a_1 \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2} \right)$$
(1)

and the specimen

$$c\left(\frac{\partial(\rho T_2)}{\partial t} + f(\rho, z)\frac{\partial T_2}{\partial z}\right) = \lambda_2(\rho)\nabla^2 T_2.$$
(2)

The boundary conditions at the envelope-mold  $(S_1)$ , specimen-die  $(S_2)$ , and specimen-plug (the die hole  $-S_3$ ) boundaries were taken into account by Newton's law. At the specimen-envelope  $(S_5)$  boundary over the entire process there is a close contact which corresponds to the conditions of joining. At the stage of extrusion the heat removal from the rod into the air  $(S_3, S_4)$  by convection and radiation was considered. Depending on the stage the functions entering into the differential equations  $f(\rho, z)$ ,  $\lambda_2(\rho)$ , and  $\rho(t)$  have the following values:

$$f(\rho, z) = \begin{cases} 0 \text{ (combustion-holding)} \\ \frac{\rho U_p z}{H_0} \text{ (pressing)}, \\ \frac{Q}{\pi R^2(z)} \text{ (textrusion)} \\ \lambda_2(\rho) = \begin{cases} \lambda_0 \text{ (combustion-holding)} \\ \lambda_0 \left(\frac{\rho(t)}{\rho_0}\right)^k \text{ (pressing)}, \\ \lambda_k \text{ (extrusion)} \\ \rho(t) = \rho_0 \left(1 - \frac{U_p t}{H_0}\right). \end{cases}$$

At the stage of combustion-holding the combustion wave with the rate  $U_b$  and temperature in the front  $T_c$  propagates over the specimen, occupying region 2. The combustion front is assumed to be flat, uniformly moving. The mean burning rate and the combustion temperature were measured in the experiment. The equation of motion of the combustion front is  $z_* = H_0 - U_b t$ .



Fig. 2. Length l (mm) of the extruded rod vs delay time  $t_d$  (sec): curve – theory, points – experiment.

Fig. 3. Length l of the product vs angle of the tapered entrance of the die  $2\alpha$  (deg).

The condition on the front is  $T(r, z_*, (H_0 - z_*)/U_b) = T_c$ . Formulating a thermal model for the stage of pressing is performed with regard to such specific features of the process at this stage as the dependence of thermophysical parameters on the density of the porous substance, varying in time, and on the movement of the upper boundary of the specimen. An equation of motion for the upper boundary of the specimen has the form  $z_* = H_0 - U_p t$ , where  $U_p$  is the velocity of the plunger of the press. At the stage of extrusion apart from the upper boundary a transient lower one appears, the equation for which is written in view of the flow rate of the substance Q in extrusion:  $z_* = -(Q/\pi r_1^2)t$ , where  $r_1$  is the radius of the guide gauge (the die hole).

The solution of the problem resulted in finding the temperature field in the specimen, the envelope, and the extruded portion of the material and in predicting the product length. When the portion of the material in the die, placed directly above its hole, lost its viability, i.e., the ability to plastic deformation, and clogged the outlet, the extrusion ceased. The obtained length – the y-coordinate of the specimen's lower boundary – was the sought length of the product.

A comparison of the model with experiment is of significance in mathematical modeling. The results of comparing calculated and experimental dependences of temperature on time at various points throughout the bulk of the specimen and the mold are presented in [3]. These curves are in good agreement.

Figure 2 shows the dependence of the product length on the delay time. We note that the results obtained are not only in qualitative but also in quantitative agreement with experiment: the deviation between the theoretical and experimental values does not exceed 10%. The undertaken comparison was made with regard for characteristics of a complex character, and the good agreement supports the adopted model and substantiates the conclusion of a dominant role of heat conduction in the processes of SHS extrusion.

A numerical analysis of isothermal levels over the specimen for various instants provides important data on the process. On the basis of constructing isothermic lines one can trace the dynamics of distribution of hot and cold masses. The most serious bottleneck in the thermal respect is the die region, in which a sharp temperature drop occurs. It turns out that regions with the lowest temperature lie not only in the vicinity of the die hole but also under the plunger of the press.

An important parameter in terms of technology is the length of the extruded portion of the specimen (completeness of extrusion). Analyzing the dependence of this characteristic on such technological parameters as the degree of deformation, the angle of the tapered entrance of the die, the pressure and velocity of the plunger of the press, and the delay time as well as on the thermal conditions on the extruded portion of the specimen (the use of guide gauges), one can distinguish the values of these parameters which are optimal in the thermal respect. Thus, the dependence of the product length on the angle of the tapered entrance of the die is shown in Fig. 3. The nonmonotonic character of this dependence permits identification of the optimal range (120-150°) for obtaining more long-dimensional products.

Such phenomena as a partial or complete clogging of the die hole by a cold material were observed in the experiments. Studying the temperature fields in the specimen at the stage of combustion-holding has shown that from a certain instant it is impossible to extrude the material since the hole is clogged by the masses of material which has lost its viability (the ability to plastic deformation). This is because of intense heat removal into the walls of the die and the gauge.



Fig. 4. Completeness of extrusion  $K_1$  (%) vs temperature of heating of the die and the gauge T, °C.

In producing heater elements based on molybdenum disilicide (a slowly burning compound) by the SHS extrusion method there was no reproducibility of the process [4]. During the analysis of the dynamics of temperature variations we elucidated the cause whose essence is in the following. The temperature range of viability of the material defined as the difference between the temperature at the end of pressing and the viable temperature for these materials is 200-500°C (for comparison, in SHS extrusion of titanium carbide hard alloys this range is equal to 900°C). Thus, forming of the material in the production of heater elements is performed close to the critical conditions when the material loses its ability to plastic deformation.

Figure 4 shows the dependence of the product length on the temperature of heating of the die and the gauge. Calculations have shown that heating of the gauge and the die within the allowable temperature range increases the length of the product (completeness of extrusion) by a factor of 1.3 in heating up to 300°C, by a factor of 1.7 in heating up to 500°C, and by a factor of 2.3 in heating up to 800°C. The recommended heating of the gauge and the die realized in the experiment made it possible not only to increase the length of the products but also to improve the quality and to increase the density of the rods.

Analysis of the Thermal Regimes of SHS Pressing. Modeling of thermal regimes of SHS pressing is also based on a stage-by-stage consideration of the process [identification of the stages: 1) combustion-delay and 2) pressing-holding under pressure] and assumes solving the system of heat-conduction equations (1) - for the mold - and (2) - for the specimen. Among the distinguishing features of the thermal model of SHS pressing is a consideration of heat transfer at the external boundaries of the press attachments. Account was also taken of the possibility of realizing of different pressing schemes - a one-sided scheme and a two-sided one [5]. The model permits determination of the temperature fields both in the compact and surrounding envelopes of heat-insulating materials (sand, asbestos cloth, datolite, etc.) and in the elements of press attachments (the mold wall, the plunger).

Comparison of the results of the model and the experiment is made in the kinetics of cooling of the mold walls (Fig. 5): during the entire process (160 sec) the theoretical and experimental curves have not only the same qualitative form but also show a good quantitative agreement.

In an effort to find efficient methods of controlling the process of SHS pressing we investigated the influence of a number of parameters on the thermal regime in the specimen and the mold. As is seen from Fig. 6, the influence of geometric parameters on the temperature in the mold is different: the initial height of the compact, unlike its diameter, substantially affects the thermal regime. For instance, realizing the one-sided scheme of pressing, by decreasing the compact height one can obtain a decrease in the thickness of the wall and abandon additional heat insulation of the material of the specimen. It is also characteristic that in two-sided pressing starting from some value of the initial height of the compact  $H_0^*$  no substantial increase of the temperature in the mold wall occurs. This result can be used in calculating press attachments for SHS pressing of large-sized products.

The analysis of time variations of temperature and comparison of temperature fields in specimens of various heights permitted the establishment of the optimal relation  $H_0/D$  from the point of view of uniformity of the thermal regime in a specimen-mold system and the smallest gradients within the specimen. It is shown that using compacts with the smallest possible  $H_0/D$  is advantageous both in the context of improving the quality of the finished product and for decreasing the thickness of the mold wall [5].



Fig. 5. Dependence of the temperature T<sup>s</sup> of the mold wall on the time of the process t, sec: 1) theory, 2) experiment.



Fig. 6. Dependence of the maximal temperature T on the inner (1) and outer (2) walls of the mold on the height of the compact (one-sided pressing) (a), the height of the compact (two-sided pressing) (b) and the diameter of the compact (c).  $H_0$ , D, mm.



Fig. 7. Temperature distribution T across the thickness of the mold wall  $\delta$  at the end of pressing.  $\delta$ , mm.

At present the method of additional heat insulation of the specimen is used, whose main function is to prevent superheating of the mold. The investigation of temperature fields in the material has shown that, with heat insulation protection of the material, the level of thermal gradients is lowered, which must favorably affect the quality of the specimen and reduce cracking. Meanwhile, with specifically chosen parameters of the process the influence of heat insulation on temperature in the mold is unimportant, which offers the possibility of abandoning additional heat insulation.

Figure 7 shows the characteristic form of the temperature drop across the thickness of the mold wall: a significant portion of the heat force is concentrated in a rather narrow region of the inner part of the mold wall, in the so-called boundary layer. The remainder of the wall works at rather moderate heat loads. The calculation of the real temperature profile in the elements of the press attachments with allowance made for specificity of SHS pressing permits a fresh approach to designing equipment for this technology. At present the design calculation of press attachments for SHS pressing is performed using the methods given in [6]. According to them, the calculation consists in choosing the dimensions of the press attachments to meet the strength conditions which require that equivalent stresses do not exceed allowable ones. Equivalent stresses are calculated correspondingly to the working pressure of the press, the chosen thickness of the mold walls, and the temperature drop across the wall thickness  $\Delta T$ , which is involved in the calculation as a constant parameter characterizing solely the material of the mold and not at all reflecting special features of the process. Such an approach is steady-state and isothermal. Since, as shown above, the maximum of heat loads falls on the narrow boundary layer, in thermal calculations it is appropriate to employ the effective wall thickness (Fig. 7), characterizing the basic region of the material with strong temperature dependence on the coordinate and the temperature drop corresponding to it. Therein lies the essence of the new nonisothermal calculation procedure, according to which the temperature drop is determined by the specific conditions of the process. As a result of strength calculations of the press attachments performed by the both procedures it appears that a nonisothermal calculation procedure enables us to substantially reduce the thickness of the mold body, which causes the weight of the press attachments to decrease, and thereby the problem of compacting large-sized products is partially solved.

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